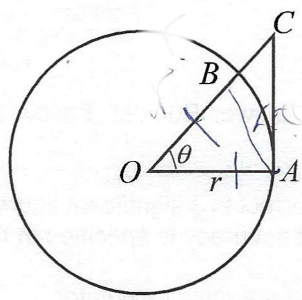
1 A cubic curve passes through the points $(2,3)$ and $(-3,-22)$. Find the equation of the curve if it has a stationary point at $(1,-6)$. [4]

2 (a) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. [3]

(b) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{6}} \sin 2\theta \cos 3\theta \, \mathrm{d}\theta$ exactly. [4]

3 (a) Given that θ is small, show that $\sec\theta \approx 1 + \frac{1}{2}\theta^2$. [2]

(b) The diagram below shows a circle, centre O and radius r, with points A and B on the circumference such that $\angle AOB = \theta$ radians, where θ is small. AC is a tangent to the circle at A and OBC is a straight line.



(i) Show that the length of chord AB can be approximated by $r\theta$. [2]

(ii) Hence show that the perimeter of triangle ABC can be approximated by $r\theta(a + b\theta)$, where a and b are constants to be determined. [4]

4 The Folium of Descartes is a curve, defined by the equation $x^3 + y^3 = 3axy$, where a is a real constant. It is given that $a \neq 0$ for this question.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{ay - x^2}{y^2 - ax}$. [2]

(b) The point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ lies on this curve. Show that the equation of the normal at this point on the curve is independent of the value of a. [2]

(c) Given that the curve has a stationary point at (pa, qa), where p and q are positive constants, find the exact values of p and q. You need not determine the nature of this stationary point. [4]

5 The curve C has equation given by $y = \dfrac{1+3x-18x^2}{9x+3}$.

(a) Without the use of a calculator, find the range of values that y can take. [4]

(b) Sketch the graph of C indicating clearly its asymptotes and stationary points. [3]

(c) State a sequence of transformations that will transform the curve C to the curve with equation $y = \dfrac{1+3(2x-1)-18(2x-1)^2}{9(2x-1)+3}$. [2]

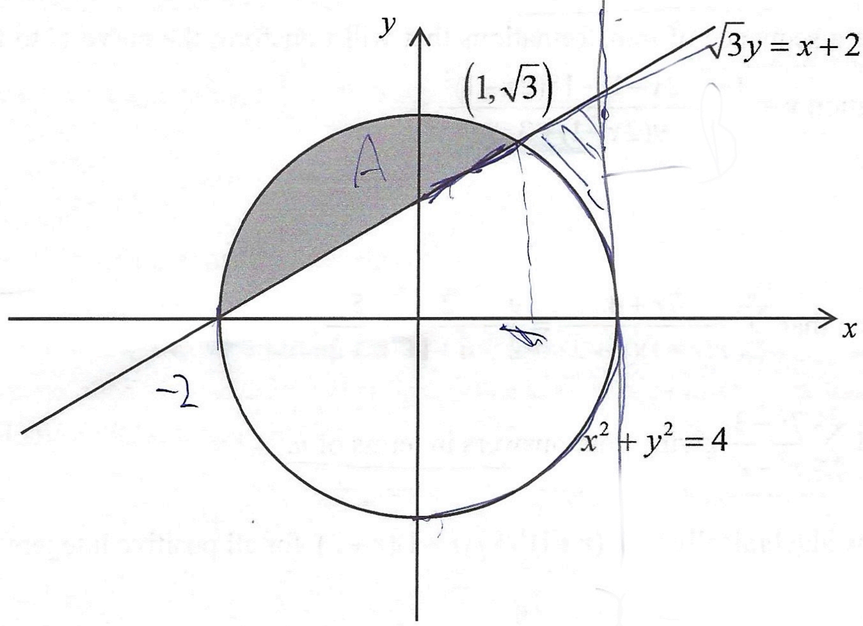
6 It is given that $\displaystyle\sum_{r=1}^{n} \frac{7r+4}{r(r+1)(r+2)} = \frac{9}{2} - \frac{2}{n+1} - \frac{5}{n+2}$.

(a) Find $\displaystyle\sum_{r=7}^{2n} \frac{7r-3}{r^3-r}$ giving your answers in terms of n. [4]

(b) Show algebraically that $(r+1)^3 > r(r+1)(r+2)$ for all positive integers r. [2]

(c) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{7r+4}{(r+1)^3} < \frac{9}{2}$. [3]

7 Curve C is a circle with radius 2 and center at the origin with equation $x^2 + y^2 = 4$. Line L has the equation $\sqrt{3}y = x + 2$. The diagram below shows the shaded region A which is enclosed between C and L.



(a) Use the substitution $x = 2\sin\theta$ to show that the area of region A can be written in the expression $\int_b^a 4\cos^2\theta \, dx - c$, where a, b and c are exact constants to be determined. Hence evaluate this area exactly. [6]

(b) The region B is bounded by the curve C, the line L and the line $x = 2$. Find the volume generated when region B is rotated through 2π radians about the y-axis. Give your answer to two decimal places. [3]

8 **(a) (i)** The ninth, fifth and second terms of an arithmetic progression are successive terms of a geometric progression with first term a and common ratio r, where $r \neq 1$ and $a > 0$. Find the value of r and deduce that the geometric series is convergent. [3]

(ii) Using the value of r found in **(i)**, find the least value of n such that the sum of all the terms after the n^{th} term of the geometric progression is less than 1% of its sum of the first n terms. [3]

(b) The sum, S_n, of the first n terms of another sequence is given by $S_n = n\ln 2^{q(n+1)}$, where q is a constant. Prove that the sequence follows an arithmetic progression. [4]

9 The line ℓ has the equation $\dfrac{x-a}{2}=y-2=\dfrac{1-2z}{b}$, where a and b are real constants. The plane π has the equation $3x-y+4z=10$.

(a) Given that the line ℓ and the plane π do not intersect, show that $a\neq\dfrac{10}{3}$ and $b=\dfrac{5}{2}$. [5]

(b) It is given that $a=1$ and $b=3$. Find the point of intersection between the line ℓ and the plane π. [3]

(c) It is given now that $a=1$, $b=\dfrac{5}{2}$.

(i) Find the distance between the line ℓ and the plane π. [2]

(ii) Determine whether the line ℓ and the origin O lie on the same side of the plane π. State an equation of the other plane that is equidistant from line ℓ and parallel to plane π. [3]

10 The function f is defined by

$$\mathrm{f}: x \rightarrow 2x-\frac{1}{2x}, \quad 0<x<2.$$

It is given that f^{-1} exists.

(a) Define f^{-1} in a similar form. [3]

(b) Sketch the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ on the same diagram. [2]

(c) The region R is bounded by the curve $y=\mathrm{f}^{-1}(x)$, $y=x$ and the y-axis. Find the exact area of R. [3]

(d) Another function g is defined by

$$\mathrm{g}: x \rightarrow \begin{cases} \dfrac{3}{2}+\dfrac{3}{3x-11} & \text{for} \quad x\leq 3, \\ \left|x-\dfrac{1}{3}x^2\right| & \text{for} \quad 3<x\leq 5. \end{cases}$$

Show that the composite function gf exists and find the range of gf. [2]

11 In a chemical reaction, compounds X and Y react to form a product. Let x and y denote the concentrations, in mol per kilolitres (mol/kL), of X and Y respectively, at time t minutes after the start of the experiment.

The initial concentrations of X and Y are given by x_0 and y_0 mol/kL.

(a) In a particular experiment, a student claimed that the rate of reaction can be modelled by the differential equation $\frac{dx}{dt} = -ax$, where a is a positive constant.

(i) Solve this differential equation, expressing x in terms of t, x_0 and a. [3]

(ii) The half-life of the reaction, denoted by $t_{0.5}$, is defined as the time taken for the concentration of X to decrease to half its initial value. Show that $t_{0.5} = \frac{\ln 2}{a}$. [2]

(b) In another experiment conducted by a chemist, the rate of the reaction is directly proportional to the product of the concentration of X and the square of the concentration of Y. It is also known that at any instance during the reaction, every 1 mol of X reacts with every 2 mol of Y, giving the equation $y_0 - y = 2(x_0 - x)$.

The initial concentrations of X and Y are 1 mol/kL and 4 mol/kL respectively.

(i) Show that $\frac{dx}{dt} = -bx(x+1)^2$, where b is a positive constant. [2]

(ii) It is given that the concentration of X is 0.5 mol/kL at the instance 1 min after the start of the experiment. Find the concentration of X at the instance 2 min after the start of the experiment, giving your answer to 3 significant figures. [6]